SAS 5.7 – Making Decisions from Cyclical Functions in Science and Economics

Recall from your science class that sound travels in waves. A wave has several important parts:

The ***crest*** is the maximum height of a wave, and the ***trough*** is the minimum height of a wave. The ***period*** is the distance between two consecutive crests or two consecutive troughs. The ***axis of symmetry*** is a horizontal line that runs exactly halfway between the crests and troughs. The ***amplitude*** is the distance between a crest or trough and the axis of symmetry.

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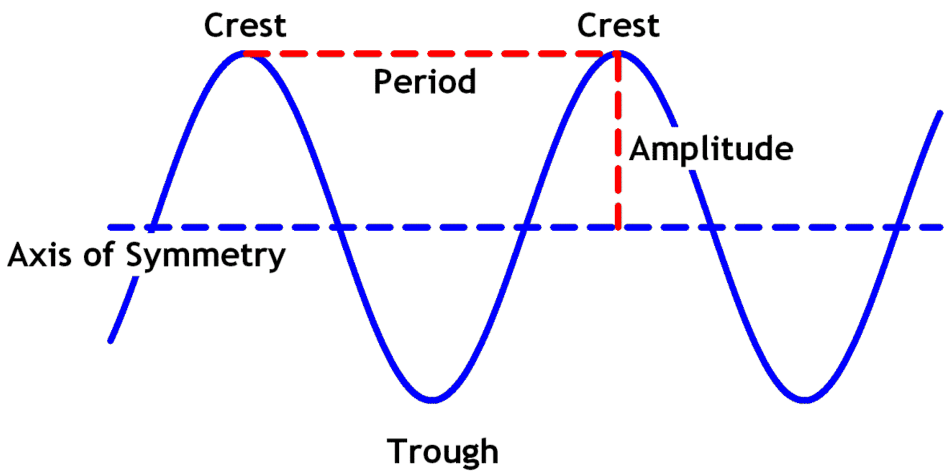

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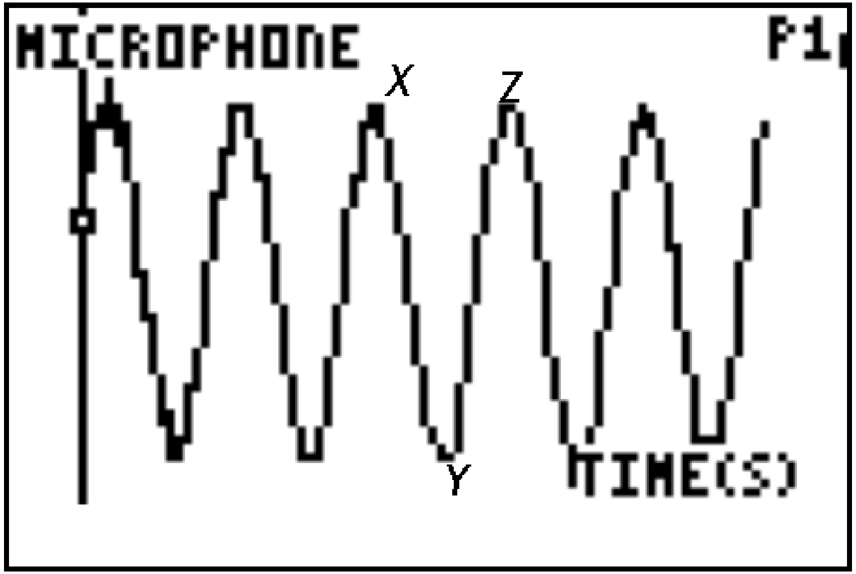
Mr. Licefi's math class used a calculator-based laboratory (CBL) and a microphone to collect the following sound data. Notice that Points *X*, *Y*, and *Z* are labeled in the graph.



X (0.0054, 6.5)

Y (0.0065, 2.5)

Z (0.0076, 6.5)

1) If *X* and *Z* each represent a crest, what is the period of the sound wave?
   (Do not forget your units!)

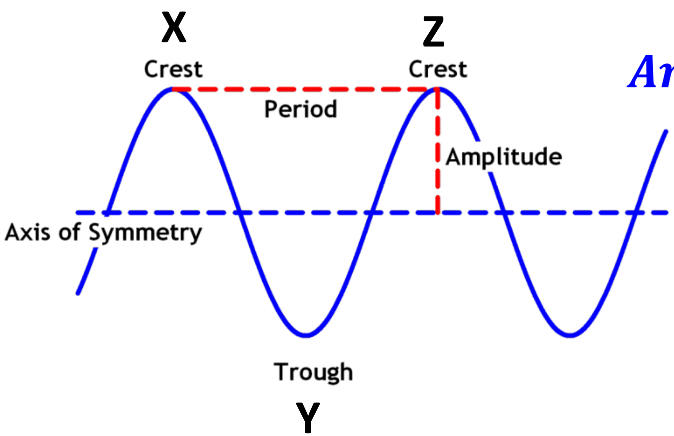
$$\boldsymbol{Period = .0076 - .0054}$$
$$\boldsymbol{= .0022\ seconds}$$

2) The frequency of a sound wave can be found by taking the reciprocal of the period. What is the frequency of this sound wave? The unit for frequency is hertz.

Since $Frequency = \frac{1}{Period}$

Then, $F = \frac{1}{.0022} = 454.55\ Hertz$

3) If Y represents a trough, what is the amplitude of the sound wave?



$$Amplitutde = \frac{Vertical\ distance\ from\ trough\ to\ crest}{2}$$

$$= \frac{6.5 - 2.5}{2} = \frac{4}{2} = \boxed{2}$$

4) In a sound wave, the frequency represents the pitch of the sound, and the amplitude represents the volume. For the sound wave that Mr. Licefi's class measured, what is the pitch and volume?

$Pitch = 454.55\ hertz$
(Frequency)

$Volume = 2$
(amplitude)

5) What amplitude is required to produce a sound wave that is twice as loud?

Double the amplitude : $\boxed{4}$

6) What are the domain and range of the function that models the sound wave?

$D:\ all\ real\ numbers$

$R:\ [2.5, 6.5]\ or\ 2.5 \leq y \leq 6.5$

7) If the sound that Mr. Licefi's class measured lasted for 8 seconds and stayed the same pitch (from Question 4), what are the domain and range of the sound wave?

$\boldsymbol{D}$: $[0, 8]$ *or* $0 \leq x \leq 8$

$\boldsymbol{R}$: $[2.5, 6.5]$ *or* $2.5 \leq y \leq 6.5$

8) Compare the domain and range for the function that models the sound wave and the domain and range for the sound wave itself. Explain any similarities or differences.

The range for the actual sound wave recorded is the same as the range for the model since they both have the same amplitude.

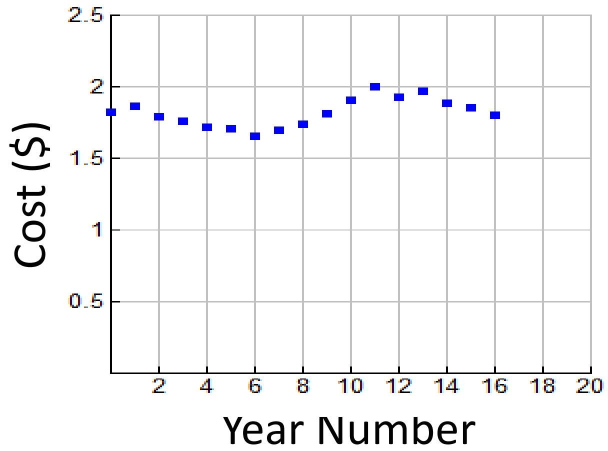
The Domains are different since the actual sound wave was only recorded for a time starting at 0 seconds and ending 8 seconds whereas, the function model continues to infinity.

Mrs. Kline's economics class was studying a data set that gives the price per pound of ground beef for the month of January from 1980 to 1996.

Year	Year Number	Cost (dollars)
1980	0	1.82
1981	1	1.86
1982	2	1.79
1983	3	1.76
1984	4	1.72
1985	5	1.71
1986	6	1.66
1987	7	1.70
1988	8	1.74

Year	Year Number	Cost (dollars)
1989	9	1.81
1990	10	1.91
1991	11	2.00
1992	12	1.93
1993	13	1.97
1994	14	1.89
1995	15	1.85
1996	16	1.80

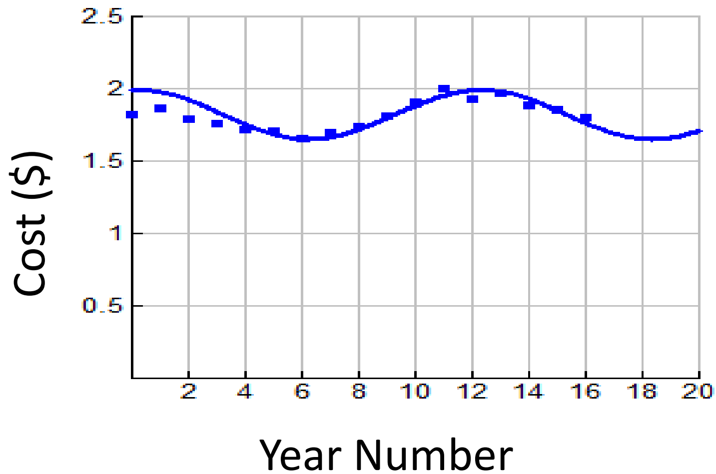
9) Use your graphing calculator to make a scatterplot of cost by year number.



10) Does the data set appear to be cyclical? Explain your reasoning.

This data does appear cyclical. The prices appear to increase then decrease and then increase then decrease again in a manner that suggests a cyclical function.

11) An economics textbook suggests that the function $y = 0.169\sin[0.52(x + 2.78)] + 1.82$ can be used to model the data approximately. Graph this function over your scatterplot to verify that suggestion. Describe the axes and scaling, and sketch your graph.



12) Recall that sine functions can be represented using the general form:

$y = \mathbf{A}\sin\left(\boldsymbol{B}(\boldsymbol{x} - \boldsymbol{C})\right) + \boldsymbol{D}$, where

- ***A*** represents the amplitude,
- ***B*** represents the angular frequency,
- ***C*** represents a factor of a horizontal translation, and
- ***D*** represents the vertical translation.

For this function, determine the values of ***A, B, C,*** and ***D***.

$\boldsymbol{A = .169\ \ (amp)}$
$\boldsymbol{B = .52\ \ (Freq)}$
$\boldsymbol{C = -2.78\ \ (h.Shift)}$
$\boldsymbol{D = 1.82\ \ (v.Shift)}$

13) Find the length of one cycle by dividing 2π by the frequency (***B***).

**** Recall from the Unit Circle that one full revolution is 2π. This means that one cycle (or period) is 2π.**

To calculate the length of one cycle (or the period) we use the formula:

$$Period = \frac{2\pi}{B}$$

$$= \frac{2\pi}{.52} = \boxed{12.08}$$

14) How well does the suggested function model the data?

For the beginning values in the data set, the function is higher than the actual values. Beginning with the fifth data point, however, the function seems to connect the values well.

15) Use the regression equation to predict the cost per pound of ground beef in January 2009.

Since 1980 is year 0, then 2009 is $2009 - 1980 = 29$

Using the given regression model we can predict in year 29 the price will be

$\boxed{\$1.70\ per\ pound}$

16) Below is a table of data from the U.S. Labor of Statistics depicting the actual cost per pound of ground beef. Use this data to determine the actual cost of ground beef in January 2009. How well did your model predict the cost of ground beef in January 2009? Why do you think the model performed this way?

Year	Cost ($)
2003	2.13
2004	2.59
2005	2.48
2006	2.61
2007	2.63
2008	2.73
2009	2.96

- **From the table we see the actual cost of ground beef in 2009 was $2.96.**
- **The model did predict the cost well at all. The model predicted $1.68, this is a difference of $1.26 per pound!**
- **Notice beginning in 2007 food prices rose quickly along with energy prices and an economic recession that began in 2008. These two events could not have been foreseen in 2003, the last year for data in the table. We can conclude the model fits the data reasonably well, but does not predict future values well.**

17) What can you say about using a cyclical model to predict values beyond a given data set?

A cyclical model that is used for a particular data set assumes that the cycle continues over time without interruption. In some cases this happens while in others, such as the price of ground beef, this assumption is not valid. Unforeseen events can disrupt behavior that is otherwise cyclical.

18) What other natural or business phenomena could be modeled using a cyclical model? How well do you think those models could predict future values?

Ocean waves can be modeled by Sinusoidal functions very well as long as the waves travel uninterrupted. This occurs when you are in deep ocean waters. However, as you get closer to land the waves will break over sandbars at which point the Sinusoidal function will cease to provide a good model.