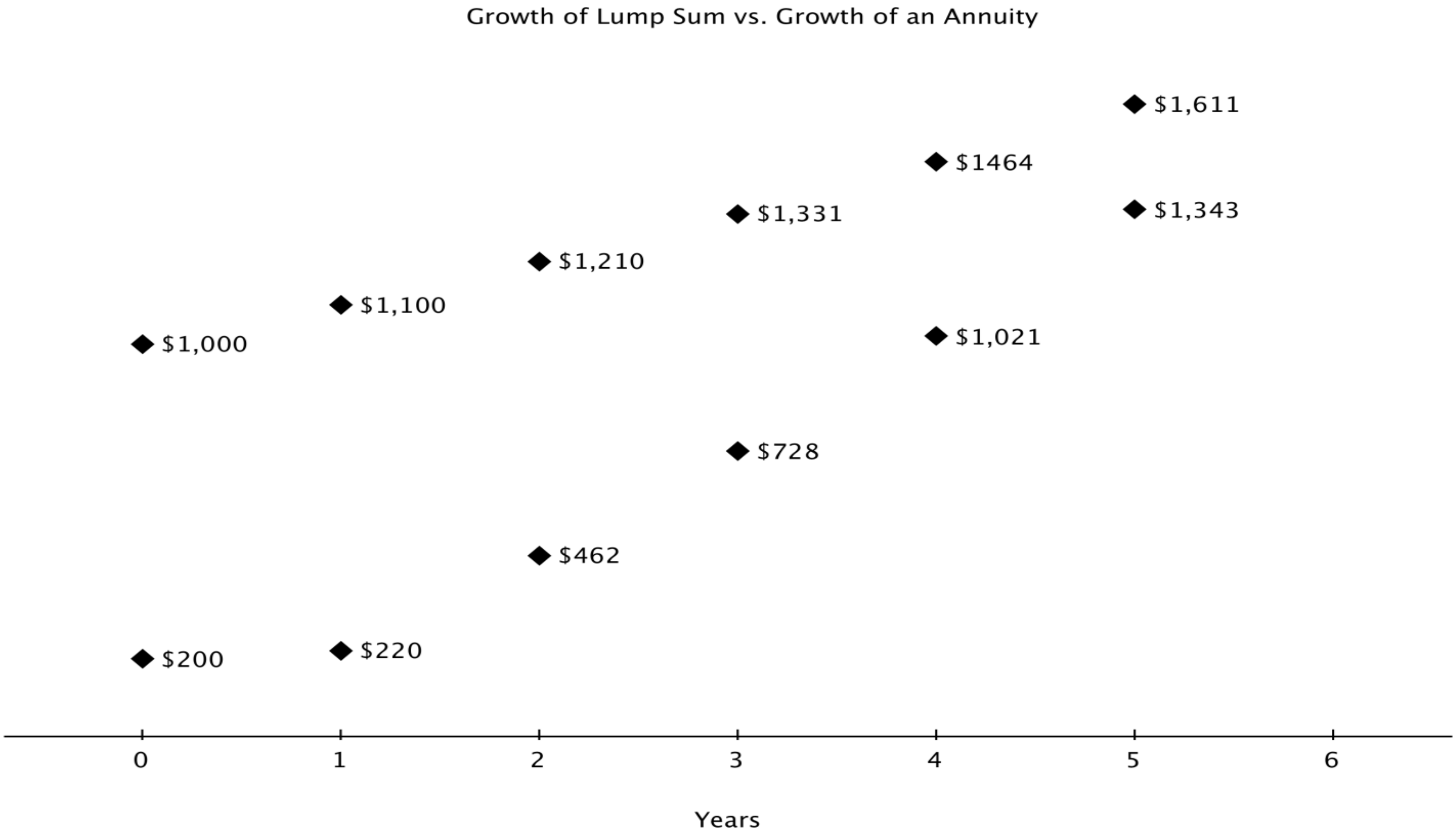
SAS 6.6 – Investing As You Go

An **annuity** is a financial product that accepts and grows funds and then, upon annuitization, pays out regular payments to the investor. Annuities are often used as retirement funds. Some annuities are funded with a lump-sum investment, while others are funded with an initial investment and additional regular deposits before retirement. What complicates the time value of money (TVM) of an annuity that you pay into is that the investment increases in value due to both compound interest and increasing principal.

The following graph shows the value of a lump-sum investment of $1,000 earning 10% compounded per year (•) versus an annuity with an initial investment of $200 earning 10% compounded per year with additional $200 deposits made each year (♦).

***note: this particular annuity has the payments being applied at the beginning of each compounding period. That changes the "order" that calculations are made.**


Growth of Lump Sum vs. Growth of an Annuity
$1,611
$1464
$1,343
$1,331
$1,210
$1,100
$1,000
$1,021
$728
$462
$200
$220
0
1
2
3
4
5
6
Years

1) How is the process different for calculating the future value of each investment?

The lump-sum investment earns compounded interest off the $1,000 principal, whereas the annuity earns interest off $200 and then adds another $200 to the principal.

2) Refer to the future-value formula in Student Activity Sheet 3. How is the process different in calculating the future value of an annuity when compared to using the future-value formula?

- **The lump-sum investment is exponential and the FV formula can be used to model it.**

- **The annuity is not continuous since there will be jumps in the graph each year when $200 is added to the principal amount. Use of a recursive model is required here.**

3) An annuity can be thought of as a series of values connected by a common ratio. What common ratio connects the values of the annuity over time shown in the graph at the beginning of this activity sheet? How is the ratio related to the problem situation?

The common ratio is 1.1. This represents the 10% interest paid (1 + 0.1) = 1.1.

4) The following formula can be used to calculate the sum of a series connected by a common ratio, such as the previous annuity example.

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Where, a1 = the first term in the series,
n = the number of terms in the series,
r = the common ratio.

Use the formula to calculate the value of the annuity described in the graph, and compare the results after five years.

The first term in the series is the balance at the end of the first year: 200 • 1.1 = 220.

$$\boldsymbol{S_5 = \frac{220(1 - 1.1^5)}{1 - 1.1} = \$1,343.12}$$

5) In Student Activity Sheet 5, you learned to use a TVM calculator to determine different variables related to TVM. In your prior work with the TVM calculator, you only considered lump-sum investments (and the payment variable was always 0). Explore using the TVM calculator to determine the future value of the $200 annuity over five years, and compare your answer with the known future value of $1,343.12. List the values you assigned to each variable and explain why.

(***Note:*** Interest is typically paid at the end of the compounding period. In this case, you make payments at the beginning of each period. Therefore, you must change the appropriate parameter from END to BEGIN.)

N	**1(5) = 5**
I%	**10**
PV	**0**
PMT	-200
FV	**??**
P/Y	**1**
C/Y	**1**

$$FV = \$1,343.12$$

Notice that we got the same result as we did when using the series summation formula,

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

By using the Payments (PMT) and the "BEGIN" parameters, the TVM calculator properly handles annuity type payments.

6) Amy is 25 years old and has attended some retirement planning seminars at work. Knowing she should start thinking about retirement savings early, Amy plans to invest in an annuity earning 5% interest compounded annually. She plans to save $100 from her monthly paychecks so that she can make annual payments of $1,200 into the annuity at the beginning of each year. Use the TVM calculator to determine the future value of the investment after 35 years.

N	1(35) = 35
I%	5
PV	0
PMT	$-1,200$
FV	??
P/Y	1
C/Y	1

$$FV = \$113,803.59$$

7) Amy seeks the advice of a financial planner, who recommends $850,000 for retirement. Will Amy's annuity plan provide the necessary funds for her retirement? If not, what could she do to increase the value of the investment at retirement? Of those actions, which does she have relative control over?

No, Amy will not have enough for retirement.

Sample response for actions:

Amy could work more than 40 years. If she works until age 65, she has 40 years to make payments into the annuity. However, this still only gives her $152,207.72.

Amy must increase the amount she saves from each paycheck to increase her annual payment to the annuity. Another option is to find an annuity that pays better interest.

8) Amy finds another annuity that accounts for **monthly** compounding and **monthly** payments. The annuity pays 6% annual interest, compounded monthly. Use the TVM calculator to determine the monthly payments made at the end of each month that Amy needs to make over 40 years to have $850,000 at the time of her retirement.

N	**12(40) = 480**
I%	**6**
PV	**0**
PMT	**??**
FV	$850,000$
P/Y	**12**
C/Y	**12**

In this example we need to set the calculator back to "END" since the payments are being made at the end of the compounding period.

$$PMT = \$427$$

This means that Amy needs to make monthly payments of about $427 to reach her retirement goal.